

Directed quantum communication

Johan Åberg,^{*} Stefan Hengl,[†] and Renato Renner[‡]
Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland

We raise the question whether there is a way to characterize the quantum information transport properties of a medium or material. For this analysis we find that the special features of quantum information has to be taken into account; quantum communication over an isotropic medium, as opposed to classical information transfer, requires the sender to direct the signal towards the receiver. As a consequence, the medium's capacity for quantum communication can be characterized in terms of how the size of the transmitter and receiver have to scale with the transmission distance to maintain quantum information transmission. To demonstrate the applicability of this concept, an n -dimensional spin lattice is considered, yielding a sufficient scaling of $\delta^{n/3}$ with the distance δ .

PACS numbers: 03.67.Hk

Introduction.— The propagation of disturbances in materials, e.g., electric pulses in a piece of metal, sound in a solid, or spin-waves in a spin lattice, can be regarded as a transmission of information. Evidently, the ‘quality’ of this information transmission is determined by the transport properties of the medium. This Letter considers transport properties from an information-theoretic perspective, or perhaps more accurately, the capacity for information transfer is regarded as a material property. To get an intuitive picture of what we aim for one can think of classical radio transmission over free space, i.e., imagine a propagation medium that is translation symmetric and isotropic (in a wide sense) and that we are in control only of limited transmitter and receiver regions. While radio transmission is typically modeled as classical information transfer over a classical medium, we consider quantum information transfer over quantum mechanical media. In this Letter we argue that quantum communication in an isotropic medium, as opposed to classical information transfer, requires the transmitter to direct the signal towards the receiver. The degree to which such a directed quantum communication can be achieved is a property of the medium. We suggest to characterize this quantum information transport property by how the size of the transmitter and receiver regions have to scale with increasing transmission distance in order to obtain quantum communication. As an illustration we use an n -dimensional spin lattice, where an upper bound to the scaling can be determined.

In the specific setting of higher-dimensional spin lattices, this investigation can be regarded as a generalization of the idea to use permanently coupled 1D spin chains for information transmission [1]. For 1D spin chains it is known that perfect state transfer can be obtained by tuning the interactions locally along the chain [2]. One could imagine this to be possible also in higher dimensions [3]. However, as we consider the ‘free space’ of a translation symmetric lattice, this excludes such local tunings.

We note that this study is related to the Lieb-Robinson (LR) bound [4]. The LR bound can be rephrased as an

upper bound on the speed of information propagation, which is determined by the Hamiltonian of the medium. Reasonably, the LR bound should limit how efficiently quantum information can be transmitted in a medium.

Quantum information transport is possible when the medium admits a non-zero quantum channel capacity. The latter measures how many qubits that can be sent reliably, averaged over many independent repetitions of a channel, assuming optimal encodings and decodings. (We consider the unassisted capacity, where, e.g., no additional classical channels are assumed.) To apply this concept we need to specify a channel, i.e., a well defined physical mapping from an input system to an output system. A channel can be set up by ‘injecting’ information from an input system A into the transmitter region of the medium. (For a concrete example in the special case of a spin lattice, see Fig. 1.) If the input system A initially is uncorrelated with the medium, then the injection and the evolution of the medium result in a quantum channel from the input A to a receiver region R . One could imagine a qualitative characterization of the medium simply by asking whether the resulting channel capacity is non-zero or not. However, the answer will depend on the sizes and distance between the transmitter and receiver. To avoid this, we rather ask how the transmitter and receiver regions have to *scale* with the transmission distance to obtain a non-zero capacity. (To use scaling as a method to get rid of unimportant details is a common approach. One example is the area law scaling of entanglement entropy, see e.g. [5].) The transmission still depends on other aspects of the information injection (and the extraction at the receiver) but the optimal scaling achievable (possibly under some constraints, e.g., a bound on the energy) can be taken as a characterization of the medium. Needless to say, the optimal scaling would in general be very challenging to determine. More realistically, we can find upper bounds (sufficient scaling) to the theoretically optimal scaling. (This is analogous to the classical setting where one in general has to settle for lower bounds on the channel capacity over a given medium.) With the purpose to obtain such scalings, we

first elucidate some necessary and sufficient conditions for a non-zero channel capacity in various settings. We begin with a simple argument, which shows that if there is too much symmetry in the system, then the quantum channel capacity is zero.

Need for symmetry breaking.— Classical signals can be copied and transmitted in all directions, e.g., in radio broadcasting, where the copying is done by ramping up the amplitude in the transmitter antenna. Since quantum information cannot be cloned [6] or broadcast [7] one might suspect that there is no quantum analogue of this. We can make this intuition more precise in terms of a symmetry argument. For this purpose we assume the medium to have some type of symmetry, and furthermore assume that the state of the medium after the injection is invariant under this symmetry, for all states of the input system A . (Since we typically imagine a localized transmitter, the symmetries would be, e.g., rotations or reflections around this region.) The symmetry generates copies of the receiver region R . If such a copy R' does not overlap with R , then they correspond to two distinct subsystems of the medium. By the assumed symmetries, R and R' will obtain the same state no matter the input A . Intuitively, the no-cloning theorem thus implies that there is no quantum information transmission from A to R . More formally, since the state of R can be reconstructed from R' , this implies that the channel from A to R is anti-degradable [8], which gives a zero quantum channel capacity [8, 9]. We can thus conclude that the symmetry makes quantum communication impossible. Hence, within the limits of our local control we should try to break the symmetry, and in this sense direct the signal to the receiver. This is in contrast to the classical case, where a similar symmetry condition may lower the efficiency, but would not prevent information transmission per se.

Need for focusing.— Next we consider an example which shows that mere symmetry breaking is in general not enough to obtain a non-zero channel capacity; the quantum signal needs to be directed in a stronger sense. Consider a medium where information is transmitted via single excitations or particles. (We do not specify whether the medium is discrete or a continuum.) In this setting one can determine a simple necessary and sufficient condition for a non-zero quantum channel capacity. We assume that the medium preserves the total number of particles, i.e., its Hamiltonian commutes with the total number operator. We furthermore assume that the medium has a vacuum state $|\nu\rangle$ that can be written as a product state $|\nu\rangle = |0_R\rangle|0_{R^c}^c\rangle$ of local zero-excitation states $|0_R\rangle$ and $|0_{R^c}^c\rangle$ in the receiver R and its complement R^c , respectively. Moreover, the single-excitation sector is spanned by states of the form $|\chi_R\rangle|0_{R^c}^c\rangle$ and $|0_R\rangle|\chi_{R^c}^c\rangle$, where $|\chi_R\rangle$ and $|\chi_{R^c}^c\rangle$ are single excitation states on R and R^c , respectively [10]. The input A is a single qubit, the medium starts in the vacuum state, and the injection can be de-

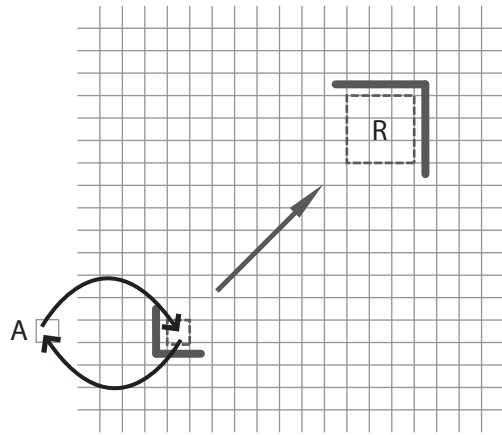


FIG. 1: To obtain a channel from a transmitter to a receiver over a spin lattice, we may use a separate spin A as an input system. To ‘inject’ this information into the lattice, we swap the input spin A with a selected spin in the lattice. A local potential barrier acts as a transmitter antenna that directs the excitation towards the receiver, where the wave packet reaches another antenna that collects the excitation into the receiver area. By considering the state in the receiver region R at a given time we obtain a channel from the input spin A to the receiver R .

scribed by a unitary operator U_I . If A is in state $|0\rangle$ then the injection does nothing, i.e., $U_I|0\rangle|\nu\rangle = |0\rangle|\nu\rangle$, while it puts a single excitation state, $|\eta_T\rangle$, in the transmitter region if A is in $|1\rangle$, i.e., $U_I|1\rangle|\nu\rangle = |0\rangle|\eta_T\rangle$. The dynamics of the lattice evolves $|\eta_T\rangle$ into a new single-excitation state $|\psi_p\rangle = \sqrt{p}|\chi_R\rangle|0_{R^c}^c\rangle + \sqrt{1-p}|0_R\rangle|\chi_{R^c}^c\rangle$, where p is the probability to find the excitation in the receiver region. If the state of the input qubit A is ρ , then the state of the receiver region R can be written as

$$\begin{aligned} \Phi_p(\rho) = & \langle 0|\rho|0\rangle|0_R\rangle\langle 0_R| + p\langle 1|\rho|1\rangle|\chi_R\rangle\langle \chi_R| \\ & + \sqrt{p}\langle 1|\rho|0\rangle|\chi_R\rangle\langle 0_R| + \sqrt{p}\langle 0|\rho|1\rangle|0_R\rangle\langle \chi_R| \\ & + (1-p)\langle 1|\rho|1\rangle|0_R\rangle\langle 0_R|. \end{aligned}$$

Effectively, Φ_p is a qubit amplitude damping channel, and for these it is known that the channel capacity is non-zero if and only if $p > 1/2$ [11]. If combined with the previous symmetry argument, we see that it is not enough to break the symmetry in order to get a non-zero capacity, but that the receiver furthermore has to collect most of the amplitude of the particle [12]. (We will later compare this with a multi-excitation transmission model.)

This threshold effect implies that for a given distance there is a minimum size of the transmitter and receiver below which the quantum channel capacity is strictly zero. In the quantum case we thus obtain a nontrivial characterization of the medium in terms of the rate at which this minimal size grows with the distance. In the classical setting one would rather expect a decaying but non-vanishing capacity, in which case the scaling would

be trivial, as fixed size antennas would yield a non-zero (albeit small) capacity for any distance.

Possibility of directed quantum communication.— To illustrate the possibility of directed quantum communication, we take a square lattice L of uniformly coupled spin-half particles that interact according to the Heisenberg XY-model

$$H = -\frac{1}{2} \sum_{\langle j,k \rangle} (\sigma_j^x \sigma_k^x + \sigma_j^y \sigma_k^y) + \sum_j (\sigma_j^z + \hat{1}_j), \quad (1)$$

where σ_j denote Pauli-matrices, and $\langle j,k \rangle$ nearest neighbor pairings. In the 1D case (allowing for varying coupling constants) this is a common model for information transfer in spin chains (see e.g. [2]). Since $[H, \sum_j \sigma_j^z] = 0$, the total number of excitations is conserved, and the ground state is a product state $|0\rangle \cdots |0\rangle$ where 0 denotes spin down. The simple dynamics of this model facilitates numerical calculation of the pick-up probability (and thus the channel capacity). Due to computational limitations we only consider the 2D case.

In Ref. [13] it was observed that a single excitation can propagate along diagonals of the 2D square lattice XY-model in a remarkably confined manner (see Fig. 7 in Ref. [13]). However, the wave packet disperses more rapidly in other directions. In other words, the pick-up probability in the receiver region and hence the channel capacity depends on the direction of propagation, similar to other transport properties. In the present calculations we consider propagation along the favored diagonals.

One can imagine several different methods to direct the excitations towards the receiver. One way is to construct local potential barriers, as depicted in Fig. 1. These potentials are obtained by adding terms of the form $w_j \sigma_j^z$ to Eq. (1), where w_j are real numbers. We use this simple type of antennas for the calculation of the dashed line in Fig. 2 (a), which gives the pick-up probability p as a function of the time t between the swap-in from A and the time when we record the state in R . As Fig. 2 (a) shows, p reaches above the critical value $1/2$ for this specific arrangement. Another method to obtain the necessary directionality is to put a suitably shaped wave packet directly on the lattice. The solid line in Fig. 2 (a) gives one example of this for a modulated Gaussian wave packet cropped to a small transmitter region. Numerical tests suggest that this method is superior compared to the above antenna construction, in terms of the needed size of the transmitter region for communication over a given distance.

Sufficient scaling.— Using the above model, with transmission along the diagonals of the lattice, we here turn to the question of how fast the transmitter and receiver have to grow with the transmission distance to obtain a non-zero channel capacity. A crucial issue is how fast a given single-particle wave-packet spreads as it propagates, and thus minimally dispersive wave-packets should

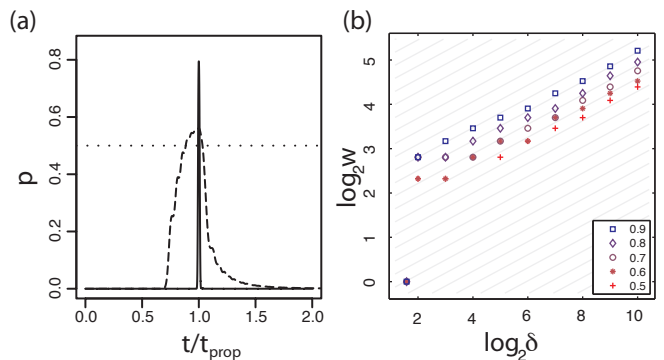


FIG. 2: (a) Pick up probability: The probability p to find the excitation in the receiver area is plotted as a function of the evolution time t in units of t_{prop} . The latter is the propagation time (different for the two graphs) that gives the maximal p . The dashed line corresponds to the setting schematically depicted in Fig. 1, with a 256×256 lattice with lossy edges, and a 20×20 receiver area. The distance between the inner corners of the antennas is 110 sites. The solid line corresponds to a 2048×2048 lattice, with 21×21 transmitter and receiver regions. The distance between the centers of these two squares is 1969 sites. In this case we have no antennas, but use as initial state a suitably modulated gaussian wave packet, cropped to the transmitter region. As seen, both cases reach above the critical value $1/2$.

(b) Scaling: With a transmitter and receiver at distance of δ sites in the lattice, we let the initial wave package be a Gaussian, modulated to travel at the maximal group velocity, and cropped to a square transmitter region with a side length that scales as $\delta^{1/3}$. For this transmission system we determine the side length w of a square-shaped receiver region needed to obtain a given pick-up probability p , as a function of δ . We plot $\log_2 w$ against $\log_2 \delta$, and repeat this for the pick-up probabilities $p = 0.9, 0.8, 0.7, 0.6, 0.5$. The lines in the background are set to the slope $1/3$.

be useful. For the 1D XY-model it was found [14] that a good choice of such wave-packets yields a pick-up probability close to 1, for transmitter and receiver regions that grow like $\delta^{1/3}$, where δ is the number of spins in the spin chain [15]. This suggests an analogous approach for the XY-model on an n -dimensional square lattice, since the evolution is decoupled along the n different dimension, which would yield a volume scaling of $\delta^{n/3}$ of the transmitter and receiver regions. This reasoning is confirmed in Fig. 2 (b) by a numerical calculation of the scaling in the 2D case. Since we have used a specific transmission system, this is an upper bound to the theoretically optimal scaling. However, restricted to the set of single-excitations, it appears reasonable to expect this result to be near optimal.

Multi-particle transmission.— One may certainly wonder to what extent the single-particle case gives a representative picture of a typical medium, where a disturbance could involve many particles. To get a hint of the effects of multi-particle signals we generalize the single-

particle transmission setup such that $|\eta_T\rangle$ is a N -particle state, rather than a single-particle state. Thus, after the evolution, the new N -particle state can be written $|\psi\rangle = \sqrt{p}|0_R^c\rangle|\chi_R^N\rangle + \sqrt{q}|\chi_{R^c}^N\rangle|0_R\rangle + \sqrt{r}|\chi\rangle$, where $|\chi_R^N\rangle$ is an N -particle state in the receiver region, $|\chi_{R^c}^N\rangle$ an N -particle state in the complement, and $|\chi\rangle$ is an N -particle state with more than zero particles in both the receiver and the complement. Here, p is the probability that we find all the particles in the receiver, q the probability that we find none in the receiver, and $r = 1 - p - q$ is the probability that we find some particles in both the receiver and its complement. The channel from the input qubit A to the receiver R can be written

$$\begin{aligned}\Phi(\rho) = & \langle 0|\rho|0\rangle|0_R\rangle\langle 0_R| + p\langle 1|\rho|1\rangle|\chi_R^N\rangle\langle\chi_R^N| \\ & + \sqrt{p}\langle 1|\rho|0\rangle|\chi_R^N\rangle\langle 0_R| + \sqrt{p}\langle 0|\rho|1\rangle|0_R\rangle\langle\chi_R^N| \\ & + q\langle 1|\rho|1\rangle|0_R\rangle\langle 0_R| + r\langle 1|\rho|1\rangle\sigma,\end{aligned}$$

where σ is a density operator with support on the orthogonal complement to the space spanned by $|0_R\rangle$ and $|\chi_R^N\rangle$. With similar techniques as for the amplitude damping channel (i.e., degradability [16] and anti-degradability [8]) one can prove that this channel has a non-zero channel capacity if and only if $p > q$. Hence, the channel capacity is non-zero if and only if the probability to pick up all the particles is strictly larger than the probability to pick up none. Like for the single-particle transmission we thus obtain a threshold effect for the channel capacity, which enables a characterization of the medium in terms of a scaling. In general quantum media, the existence and characterization of this type of threshold effects is an open question.

Conclusions.— We have found that quantum communication requires us to direct signals. We characterize the ability of media to sustain directed quantum communication in terms of the scaling of the transmitter and receiver needed to maintain a non-zero quantum channel capacity. For single-particle transmission in an n -dimensional Heisenberg XY-model, a scaling of $\delta^{n/3}$ is sufficient.

To directly determine the optimal scalings of general physical media, e.g., solid state systems or optical lattices, appears rather challenging. A more realistic scenario would be to estimate these in terms of other material properties. For this it would be important to approach more realistic settings and include, e.g., Anderson localization, thermal noise, and decoherence.

In this Letter we have made the tacit assumption that a sequence of transmissions can be described as independent and identically distributed (iid) repetitions of a single transmission. If the medium in some sense relaxes back to its initial state after each transmission, this approximation is justifiable, as the scaling does not take into account the time it takes to transmit signals, and we thus can make sufficient delays between subsequent transmissions. However, if one would wish to determine the transmission per time unit, rather than per chan-

nel use, the iid assumption may not be useful, e.g., as the number of excitations in the medium potentially increases for rapidly repeated transmissions. While the iid assumption is natural for many information theoretic tasks, it appears to be less suitable in various physics contexts (e.g., for entanglement in critical systems [17]). For the information transmission problem in particular, and the application of information theoretic tools to physics in general, it thus appears desirable to use formalisms that do not require the iid assumption [18].

We acknowledge support from the Swiss National Science Foundation (SNF), grant nos. 200021-119868 and 200020-135048, and from the European Research Council (ERC), grant no. 258932.

* Electronic address: jaaberg@phys.ethz.ch

† Electronic address: hengl@phys.ethz.ch

‡ Electronic address: renner@phys.ethz.ch

- [1] S. Bose, Phys. Rev. Lett. **91**, 207901 (2003); S. Bose, Contemporary Physics **48**, 13 (2007).
- [2] M. Christandl *et al.* Phys. Rev. A **71**, 032312 (2005).
- [3] A. Casaccino *et al.* arXiv:0904.4510.
- [4] E. H. Lieb and D. W. Robinson, Comm. Math. Phys. **28**, 251 (1972).
- [5] M. B. Plenio, J. Eisert, J. Dreißig, and M. Cramer, Phys. Rev. Lett. **94**, 060503 (2005).
- [6] W. Wothers and W. Zurek, Nature **299**, 802 (1982).
- [7] H. Barnum *et al.* Phys. Rev. Lett. **76**, 2818 (1996).
- [8] F. Caruso and V. Giovannetti, Phys. Rev. A **74**, 062307 (2006).
- [9] A. S. Holevo, Problems of Information Transmission, 2008, **44** 171 (2008).
- [10] One can relax these assumptions. The vacuum does not have to be a product state, and it is essentially enough if the single-excitation sector is spanned by states that can be generated from the vacuum (and removed again) coherently, via local operations.
- [11] V. Giovannetti and R. Fazio, Phys. Rev. A **71**, 032314 (2005).
- [12] If we allow for an assisting flow of classical information from the receiver back to the transmitter, then one can show that quantum communication is possible if $p > 0$.
- [13] O. Mülken, A. Volta, and A. Blumen, Phys. Rev. A **72**, 042334 (2005).
- [14] T. J. Osborne and N. Linden, Phys. Rev. A **69**, 052315 (2004); H. L. Haselgrove, Phys. Rev. A **72**, 062326 (2005).
- [15] For single excitations, the Heisenberg model in [14] is equivalent to the Heisenberg XY-model we use.
- [16] I. Devetak and P. W. Shor, Comm. Math. Phys. **256**, 287 (2005).
- [17] J. Eisert and M. Cramer, Phys. Rev. A **72**, 042112 (2005).
- [18] D. Kretschmann and R. F. Werner, Phys. Rev. A **72**, 062323 (2005); A. Bayat *et al.* Phys. Rev. A **77**, 050306(R) (2008); V. Giovannetti, D. Burgarth, and S. Mancini, Phys. Rev. A **79**, 24, 012311 (2009); N. Datta and R. Renner, IEEE Trans. Inf. Theor. **55**, 2807 (2009).